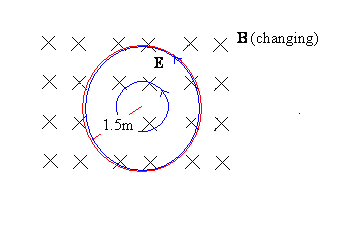
Time-dependent Fields Problems

**Problem**

Let’s go back to our example. Suppose we increase the magnetic field strength in the area from 0 to 10T in a span of 5s. What will be **E** field strength at a radius of 1.5m from the center of the center of the magnetic field distribution?

**Solution**

To find out, we draw a circle of radius 1.5m about the center of the **B** distribution.



and now we apply the rule, going around the loop in a counter-clockwise direction, say.



Therefore,



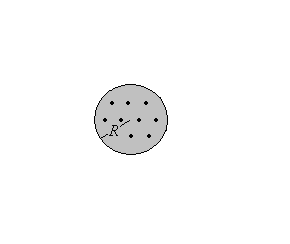
So the strength of the field is 1.5N/C. Now the magnetic flux is changing by becoming more and more negative; so it is changing in the negative direction and therefore we would point our thumb downward. Then our fingers would be curling CW. Now flip/reverse your hand. Your fingers now point in the direction of the induced **E**. So we have,



as shown.

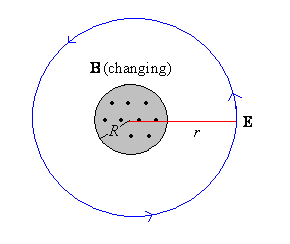
**Problem**

Let’s consider another example, where the magnetic field is confined to a circular region of radius R, and suppose that it is coming out of the page. What is the magnitude and direction of the electric field induced at a radius r > R?



**Solution**

Suppose the field changes at a rate of |dB/dt|, and is decreasing in strength, moreover. Well let’s label this,



This is how we know the direction of **E**. The flux is coming out of the page, but it is decreasing. So the flux is going changing towards the negative direction. So we point our thumb in the direction of the changing flux; that is we point it downward. Now Faraday’s law tells us to flip our hand, so now our thumb points upward, and our fingers curl CCW – the direction of **E**. What is the magnitude of **E**? We use Faraday’s law again, going around the field line depicted.



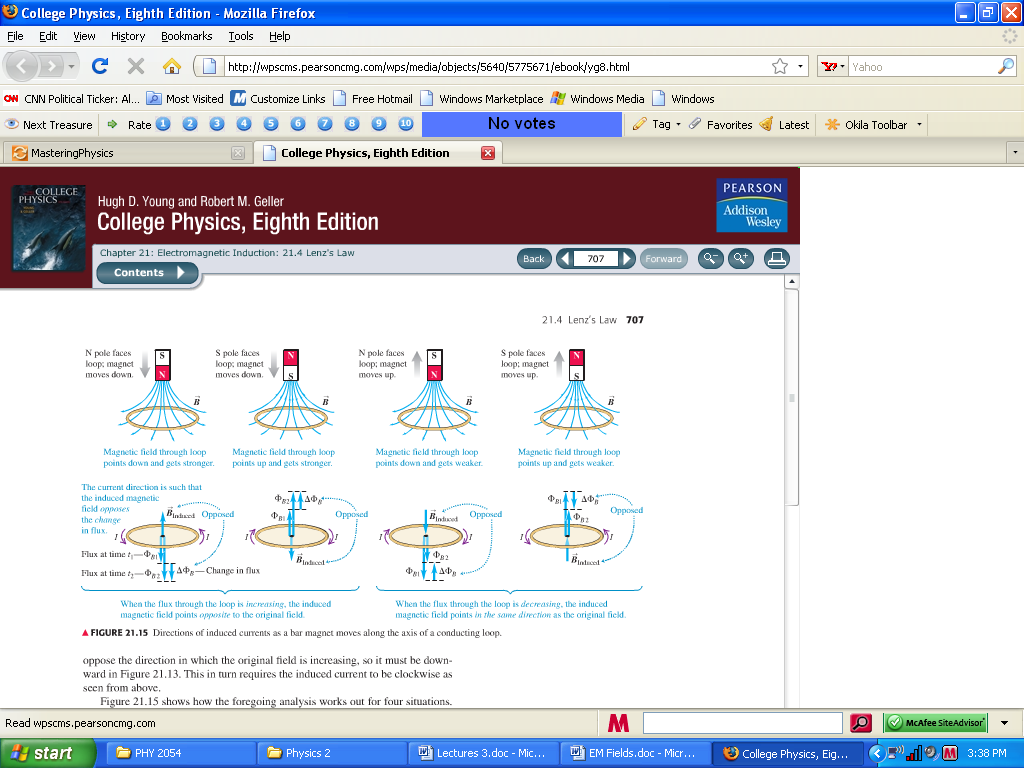
So we have,



So note how the E field strength decreases as 1/r as we recede from the region of the **B** field. Also note how the strength of E is proportional to the rate of change of **B**.

**Problem**

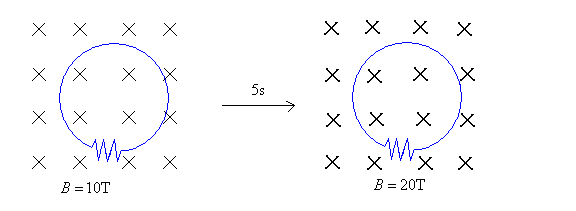
Consider the following set up described in the book. We have a wire loop, and we place it in a magnetic field oriented as shown.



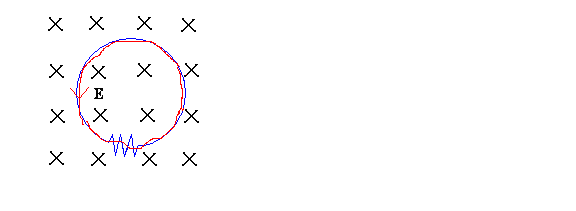
Suppose the loop as a radius of 10cm, and we move the bar magnet down closer to the loop so that the magnetic field strength increases from 10T to 20T in a span of 2s. If the wire has a resistance of 5Ω, what will be the current in the wire?

**Solution**

Well, a top down view would be as follows.



The magnetic flux through the loop changes since the strength of the field changes. Consequently, and electric field will be induced, which will go through the loop. The direction of **E** is determined from Faraday’s law. We point our thumb in the direction of increasing flux – downward. Then reverse the direction (b/c the minus sign in the RHS of the equation tells us to). Our thumb then points up, and our fingers curl CCW. So the direction of **E** is CCW.



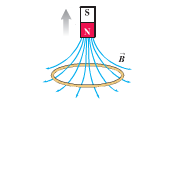
The direction of **E** is the direction of **I**. So I circulates CCW as well. As for the magnitude of I, we use Faraday’s law to get the potential difference,



and then Ohm’s law to get the current,



**Question 6.** Consider the following set up. We have a wire loop, and we place it in a magnetic field oriented as shown. Suppose the loop as a radius of 10cm, and we move the bar magnet away from the loop so that the magnetic field strength in the loop decreases from 10T to 5T in a span of 3s. If the wire has a resistance of 5Ω, what will be the induced E field and current in the wire?



The induced field is given by:



So the field is given by:

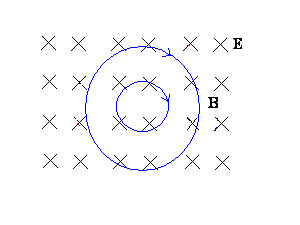


And the current induced will be I = ΔV/R,



**Problem**

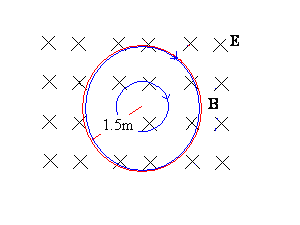
Let’s go back to our example.



Suppose we increase the electric field strength in the area from 0 to 10N/C in a span of 5s. What will be **B** field strength at a radius of 1.5m from the center of the center of the magnetic field distribution?

**Solution**

To find out, we draw a circle of radius 1.5m about the center of the **E** distribution.



and now we apply the rule, going around the loop in a counter-clockwise direction, say.



Therefore,



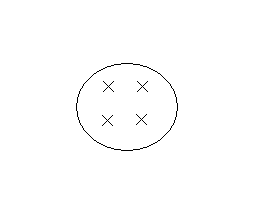
So the strength of the field is 1.67×10-17 T. Now the electric flux was increasing downward since E was increasing downward, and therefore we would point our thumb downward. Then our fingers would be curling CW - the direction of the induced **B**. So we have,



as shown.

**Problem**

Consider a wire loop of radius 5cm whose area is completely filled with a magnetic field in the direction shown. If the field strength increases from 15T to 20T in 5s, what will be the strength and direction (CW or CCW) of the **E** field induced in the loop?



**Solution**

The area of the loop is: A = π(0.05)2 = 7.85×10-3m2. We calculate the field by,

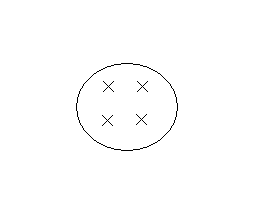


Once again, ignore the negative sign. To determine direction we point our thumb in the direction of changing flux and reverse it. Flux changes in the **z** direction since it goes from more negative to less negative. So point our thumb out of the page. Reverse it, and fingers curl CCW,



**Problem**

Suppose we use the same set up as in problem 6, but this time we fill the area with an electric field. If the field strength decreases from 50N/C to 10N/C in 3s, what will be the strength and direction (CW or CCW) of the **B** field induced in the loop?



**Solution**

This time we have,



As for the direction, the direction of increasing magnetic flux is downwards, and so pointing your right thumb downwards, we get that field curls CW. So the answer is:

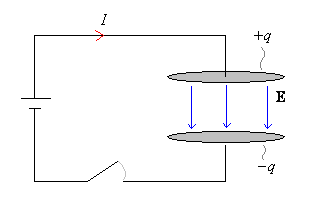


**Problem**

Let’s consider another example. Consider the following circuit, a parallel plate capacitor (radius R) connected to a battery. We know that the B field a distance r away from the long straight wires in the circuit is B = μ0I/2πr. But what is the B between the capacitor plates? Does the fact that there is no physical current running between the plates mean that there is no magnetic field there?

**Solution**

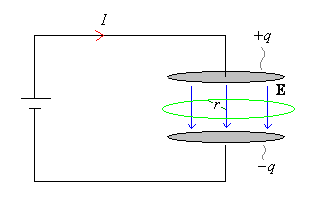
Well, while it is charging, a current I will flow through the circuit, depositing charge q on the plates. This will set up an electric field between the plates shown below.



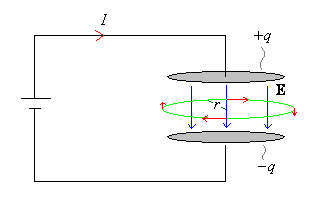
Now the electric field between the plates will depend on the charge on the plates. You may recall that the field set up between the plates of a parallel plate capacitor is:



where A is the area of a capacitor plate. Now since q will be changing with time (as the plates charge up), so too will E. The changing **E** will induce a magnetic field. What is this **B** in between the plates? Whatever **B** is, it should be cylindrically symmetric. So we draw an Amperean curve (circle) around the **E** field lines, with radius r > R.



Using the Maxwell-Ampere law to determine the direction, we point our thumb in the direction of increasing flux. The flux is going downward, and since E will be increasing, the flux will increase downward. Therefore we point our thumb downward – our fingers then point CW. So the direction of the B field lines will be CW around the green circle.

  
Now we’ll look for the strength of the induced **B** at the radius r. We use the Maxwell-Ampere law again,



Note that the area, A, appearing in the flux equation is the area of the capacitor plates. This is because **E** doesn’t exist outside of the capacitor plates (roughly) so the area of the flux is just the area of the capacitor plates. So we have,



Now what is dE/dt? Well, recalling E = σ/ε0, we have,



Now recall that the current in the wire is simply.



and so we have,



Filling this into our formula for B we have,



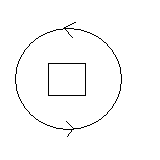
So the field is simply,



which should look familiar as the field due to a wire carrying a current I. What this tells us is that the magnetic field doesn’t care that there is an obstruction in the circuit (the capacitor plates); it sets up a **B** field just as if the current were actually going between the plates.

**Problem**

A solenoid (the outer circle) has a radius of 5 cm, turns, and a length (which goes into the page) of 10cm. The current in the solenoid is increasing at a rate of 4.0 A/s in the direction shown. A conducting square with side length 3 cm is placed at the center of the solenoid. Determine the direction and magnitude of the induced ΔV in the square.



**Solution**

The formula for the magnetic field in the middle of the solenoid is:



The area inside the square loop is A = (0.03m)2 = 9×10-4 m2. The induced emf is given by,

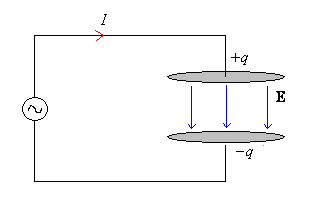


The direction is given by the rule. Since B is pointing up, and I is increasing, the flux is increasing up. Point thumb that way, reverse, and we see E goes CW.



**Problem**

A capacitor, with circular plates of radius R = 5cm, is hooked up to an AC battery. Let the charge on the capacitor be given by the equation q(t) = 30μC ∙ sin(60πt). As you might recall from early in the semester, this gives rise to an electric field inside the capacitor given by E(t) = σ(t)/ε0, where σ(t) is the charge per unit area on a plate. Now since E is changing with time, it will induce a magnetic field, B. What is the strength and direction of circulation of B at a radius r = 2cm, at time t = 0?



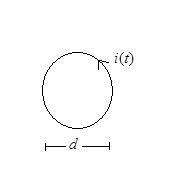
**Solution**

First we need E(t). This is E(t) = σ(t)/ε0 = q(t)/Aε0 = 30×10-6 sin(60πt)/[π(0.052)∙(8.85×10-12)] = 4.32×108 N/C sin(60πt). Then using Maxwell’s equation:



**Problem**

A wire is wrapped into a solenoid of length 20cm, diameter 5cm, and the number of turns 135. The wire is attached to a battery with a variable emf and current is run through the wires. This current is given by the formula *i(t) = 5t2 – 6t.* What is the magnitude and direction (CW or CCW) of the induced electric field inside the solenoid at a radius r = 2cm, and t = 17s? A top down view is shown below.



**Solution**

Well the field inside the solenoid is given by the formula B = μ0ni. According to Faraday’s law, the induced electric field along the radius r = 2cm is given by:

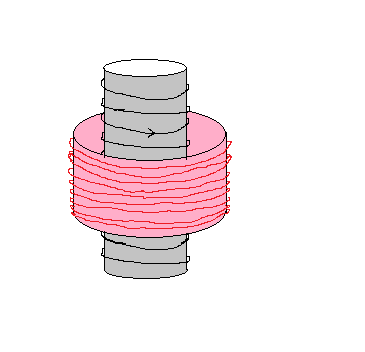


Filling in our values we get:



The negative value indicates a CW direction. But the other way to get this is to recognize that at time t = 17s, B is positive and dB/dt is also positive. So this means that the field is increasing upwards. The RHR tells us to point our thumb in this direction, then flip our hand. So our fingers are then curling in the CW direction.

**Question 11.** An inner solenoid with 100 turns wrapped around a 5cm long, 1cm radius cylinder is surrounded by an outer solenoid with 200 turns wrapped around a 2cm long, 3cm radius cylinder. The wire used to wrap the outer cylinder is made of copper (resistivity = 18nΩ∙m), and has a 1.5×10-5m2 cross section area. If the current running through the inner solenoid is circulating counter-clockwise with amplitude Iin(t) = 70e-3t, what is the magnitude of the current induced in the outer solenoid as a function of time? Which way is it circulating (clockwise, or counter-clockwise)?



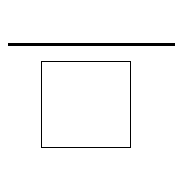
The induced emf will be:



The resistance of the outer wire is R = ρℓ/A. Now ℓ is Nout∙2πrout = (200)(2π∙0.03) = 37.7m, and so R = 0.045Ω. Therefore Iout(t) = ΔVout/R = 0.033e-3t/0.045 = 0.73e-3t. And moreover, it circulates counterclockwise.

**Question 5.** The power lines that run through your neighborhood carry alternating curents that reverse direction 120 times per second.  As the currrent changes, so does the magnetic field around a line.  Suppose you wanted to put a loop of wire up near the power line to extract power by 'tapping' the magnetic field.  Sketch a picture of how you would orient the coil of wire next to a power line to develop the maximum emf in the coil.

You would want to orient it so tha the plane of the loop is coincident with the plane of the wire, like this:



**Question 6.** A TMS (transcranial magnetic stimulation) device creates very rapidly changing magnetic fields. The field near a typical pulsed-field machine rises from 0 to 3T in 150 μs. Suppose a technician holds his hand near the device so that the axis of his 2.5 cm-diameter wedding band is parallel to the field. If the band is gold with a cross-section area of 5.0 mm2, what is the induced current? Assume the band is of jeweler's gold and its resistivity is 1.32 nΩ∙m.

The current is given by I = ΔV/R. ΔV is given by Faraday’s law:



The resistance, R, is given by:

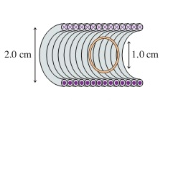


and so the current will be:



and the direction will be CCW.

**Question 11.** A 2cm diameter solenoid has 250 turns, and is 31cm long. Suppose it is hooked up to a battery which generates a current in the solenoid given by i(t) = 20sin(100t). Frodo places the ‘one ring’ (which has diameter 1cm) inside the solenoid. The ring, being of exceptionally high purity, has a small resistance R = 84nΩ. So what maximum current will be generated inside the ring? (The idea is the generate a large enough current to melt it of course)

****

Current inside the ring will be given by:



Imax would therefore be given by:



**Question 12**. A patient is placed into a solenoid (MRI machine) that is 50cm in diamter and 1.3m long. If the solenoid as 450 turns and carries a current of 95A, what energy is stored inside the magnetic field generated by the MRI?

PE = (1/2)LI2 = (1/2)(ℓμ0n2A)I2 = (1/2)(1.3)(4π×10-7)(450/1.3)2π(0.25)2(95)2 = 173 J.